Graph: $f(x) = \frac{4y^2}{x^2 - 9}$

1. Symmetry: $f(-x) = \frac{4(-x)^2}{(-x)^2 - 9} = \frac{4x^2}{x^2 - 9} = f(x) \Rightarrow$ even function (symmetric about $y$-axis)

2. $y$-intercept: $f(0) = \frac{4(0)^2}{0^2 - 9} = 0$, so point $(0,0)$ is on the graph of $f(x)$

3. $x$-intercepts: $4y^2 = 0 \Rightarrow y^2 = 0 \Rightarrow x > 0$ (confirms point (0,0))

4. Vertical asymptotes: $x^2 - 9 = (x+3)(x-3) = 0$ (these are the values of $x$ that cause $f(x)$ to be undefined)

5. Horizontal asymptotes: The order of the numerator and denominator are the same, so the horizontal asymptote is $y = \frac{4}{9}$

6. Draw vertical and horizontal asymptotes on the graph and use mathematical reasoning skills to sketch the graph.

$$f(x) = \frac{-2}{x^2 - x - 2}$$

1. Horizontal asymptote is $y = 0$ because the degree of the numerator is less than the degree of the denominator.

2. Find vertical asymptotes (where the denominator equals zero)

$$x^2 - x - 2 = (x-2)(x+1) = 0 \Rightarrow$$ asymptotes are at $x = 2$ and $x = -1$

3. $x = 2^+$; $f(x) \to -\infty$

4. $x = 2$; $f(x) \to -\infty$

5. $x = 1$; $f(x) \to -\infty$

6. $x = -1$; $f(x) \to \infty$

7. $x \to -\infty$, $f(x) \to 0$ for negative values

8. $x \to \infty$, $f(x) \to 0$ for positive values
\( f(x) = \frac{x^2 - x + 1}{x - 1} \)

1. **Vertical asymptote**: The vertical asymptote is at \( x = 1 \).

2. **Horizontal asymptote**:

   \[
   \lim_{x \to \infty} \frac{1}{x - \frac{1}{x}} = \frac{0}{1} = 0
   \]

   The horizontal asymptote is the line \( y = 0 \).

3. **\( y \)-intercept**:

   \[
   f(0) = \frac{0^2 - 0 + 1}{0 - 1} = -1
   \]

4. **\( x \)-axis intercepts**:

   Note, since \( x^2 - x + 1 \) has no real roots.

5. **Asymptotic behavior**:

   - \( x \to 1^- \), \( f(x) \to -\infty \)
   - \( x \to 1^+ \), \( f(x) \to 0 \)
   - \( x \to \infty \), \( f(x) \to \infty \)
   - Since \( f(x) = \frac{y - x}{x - 1} \)
   - \( x \to -\infty \), \( f(x) \to -\infty \)