4. Objective function: \( z = 30x + 45y \)

We must evaluate the objective function at all 4 corners of the shaded region.

<table>
<thead>
<tr>
<th>Critical Point</th>
<th>( z )</th>
<th>( 30(0) + 45(9) = 405 )</th>
<th>( 30(0) + 45(0) = 0 )</th>
<th>( 30(3) + 45(0) = 90 )</th>
<th>( 30(4) + 45(4) = 300 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,9)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Max is 405</td>
</tr>
<tr>
<td>(0,0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Min is 0</td>
</tr>
<tr>
<td>(3,0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4,4)</td>
<td></td>
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</tr>
</tbody>
</table>

Max is 405
Min is 0

14. \( z = 5x + 6y \) (Objective function)

Constraints:

(a) \( x \geq 0, y \geq 0 \) \([x \text{ and } y \text{ axes}]\)

1. \( 2x + y \geq 10 \) \( \rightarrow y = -2x + 10 \) graph these equations

2. \( x + 2y \geq 10 \) \( \rightarrow y = -\frac{1}{2}x + 5 \)

3. \( x + y \leq 10 \) \( \rightarrow y = -x + 10 \)

Find the region of solution (A, B, C, D or E)

Pick a point in each region until one is found that satisfies all of the inequalities 1 through 3.

Region A: pick (0,0).
- Satisfies only Inequality 1, so Region A is not the solution.

Region B: pick (1,5).
- Satisfies only Inequality 3 and 5, so Region B is not the solution.

Region C: pick (5,5).
- Satisfies all inequalities, so Region C is the solution region.

(b) The corners of the solution region are (10,0), (0,10) and the intersection of lines 1 and 2. Solve these inequalities simultaneously:

\[-\frac{1}{2}x + 5 = -2x + 10\]
\[-\frac{1}{2}x - 2x = 10 - 5\]
\[\frac{3}{2}x = 5\]
\[x = \frac{10}{3}\]

Sub into 1:
\[y = -2 \left(\frac{10}{3}\right) + 10\]
\[y = \frac{30 - 20}{3} \rightarrow y = \frac{10}{3}\]
(Continued)
14. (cont'd)
(b) continued

We must evaluate the objective function for the critical points:

\[
\begin{array}{c|c|c}
\text{corner} & \quad Z = 5x + 6y \quad & \text{evaluated} \\
(10,0) & 5(10) + 6(0) = 50 & \quad (a) \max = 50 \\
(0,10) & 5(0) + 6(10) = 60 & \quad (b) \quad \max = 60 \\
(10, \frac{10}{3}) & 5\left(\frac{10}{3}\right) + 6\left(\frac{10}{3}\right) = \frac{110}{3} = 36 \frac{2}{3} & \quad (c) \quad \max \text{ is } 36 \frac{2}{3}
\end{array}
\]

NOTE: Expect a very similar question on the exam!

20. Objective function: \[ Z = 30,000A + 20,000B \quad (A = \text{American plane}, \quad B = \text{British plane}) \]

Constraints:

\[ \begin{align*}
A + B & \leq 44 \\
16A + 8B & \leq 512 \\
9000A + 5000B & \leq 330,000
\end{align*} \]

These reduce to:

\[ \begin{align*}
A + B & \leq 44 \quad \rightarrow \quad B = -A + 44 \\
2A + B & \leq 64 \quad \rightarrow \quad B = -2A + 64 \\
9A + 5B & \leq 330 \quad \rightarrow \quad B = - \frac{9}{5}A + 66
\end{align*} \]

Graph Eq. (1), (2), and (3):

Now, pick points in different regions until a point is found that satisfies all three constraints.

The point (10,10) satisfies all constraints, so the solution region is the shaded part of the graph.

Solve Eq. (1) and (3) simultaneously to find where they intersect:

\[ \begin{align*}
-A + 44 & = -2A + 64 \\
A = 20 & \quad \rightarrow \quad B = 44 - 20 = 24
\end{align*} \]

Trying all the corners of the solution region, we see that the max occurs with A = 20 and B = 24.