**Combining Functions**

\[(f + g)(x) = f(x) + g(x) \quad D_{f+g} = \{x \mid x \in D_f \text{ and } x \in D_g\}\]

\[(f - g)(x) = f(x) - g(x) \quad D_{f-g} = \{x \mid x \in D_f \text{ and } x \in D_g\}\]

\[(f \cdot g)(x) = f(x) \cdot g(x) \quad D_{f \cdot g} = \{x \mid x \in D_f \text{ and } x \in D_g\}\]

\[\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad D_{\frac{f}{g}} = \{x \mid x \in D_f, x \in D_g \text{ and } g(x) \neq 0\}\]

**Combining Functions**

If \(f(x) = \sqrt{4-x}\) and \(g(x) = \frac{1}{x}\), find \((f + g)(x)\) and state its domain.

\[(f + g)(x) = \sqrt{4-x} + \frac{1}{x}\]

\(D_f = \{x \mid x \leq 4\}\)

\(D_g = \{x \mid x \neq 0\}\)

\(D_{f+g} = \{x \mid x \leq 4 \text{ and } x \neq 0\}\)

\(D_{f+g} = (-\infty,0) \cup (0,4]\)

**Composite Functions**

If \(f(x) = \sqrt{x+3}\) and \(g(x) = x - 2\), find \(\left(\frac{f}{g}\right)(x)\) and state its domain.

\[\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+3}}{x-2}\]

\(D_f = \{x \mid x \geq -3\}\)

\(D_g = \{\text{all real numbers}\}\)

\(D_{\frac{f}{g}} = \{x \mid x \geq -3 \text{ and } x \neq 2\}\)

\(D_{\frac{f}{g}} = [-3,2) \cup (2,\infty)\)

---

A function is a machine that transforms numbers in its Domain into numbers in its Range.

\[f(x) = x^2 - 1\]

\[g(x) = x + 3\]
A function is a machine that transforms numbers in its Domain into numbers in its Range.

\[ f(x) = x^2 - 1 \]

\[ g(f(x)) = f(x) + 3 = x^2 - 1 + 3 = x^2 + 2 \]

\[ (g \circ f) = g(f(x)) \]

\[ g(x) = x + 3 \]

\[ f(g(x)) = (g(x))^2 - 1 = (x + 3)^2 - 1 = x^2 + 6x + 9 - 1 = x^2 + 6x + 8 \]

\[ (f \circ g) = f(g(x)) \quad (g \circ f)(x) = x^2 + 2 \]

\[ (f \circ g)(x) = x^2 + 6x + 8 \]

Domain of a Composition

- Domain of \( g \)
- \( x \)
- \( f(\cdot) \)
- \( g(\cdot) \)
- Range of \( g \)

Domain of \((f \circ g)(x)\)

- Domain of \( f \)
- \( x \)
- Range of \( f \)

Composing Functions

If \( f(x) = \sqrt{x + 3} \) and \( g(x) = x - 2 \), find \((f \circ g)(x)\) and state its domain.

\[ (f \circ g)(x) = f(g(x)) = \sqrt{(x-2)+3} = \sqrt{x+1} \]

\[ D_g = \{ \text{all real numbers} \} \]

\[ D_f = \{ x \mid x \geq -3 \} \]

\[ g(x) \geq -3 \text{ if } x - 2 \geq -3 \text{ or } x \geq -1 \]

\[ D_{f_{g}} = \{ x \mid x \geq -1 \} \]

\[ D_{f_{g}} = [-1, \infty) \]
Composing Functions

If \( f(x) = \sqrt{x} \) and \( g(x) = x^2 - 4 \), find \( (f \circ g)(x) \) and state its domain.

\[
(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{x^2 - 4}
\]

\( D_f = \{ x | x \geq 0 \} \)

\( D_f \cdot g \cdot \{ x | x \geq 0 \} \)

Solve \( g(x) \geq 0 \)

\[
\begin{align*}
x^2 - 4 & \geq 0 \\
(x - 2)(x + 2) & \geq 0 \\
x & \leq -2 \text{ or } x \geq 2
\end{align*}
\]

\( D_{f \cdot g} \cdot \{ x | x \leq -2 \text{ or } x \geq 2 \} \)

Decomposing Functions

It is sometimes helpful to see a complicated function as a composition of simpler functions.

Transformations

- \( k(x) = 2x^2 + 3 \)
- \( f(x) = x^2 \) Standard Parabola
- \( g(x) = 2x \) Vertical Stretch
- \( h(x) = x + 3 \) Vertical Shift

\[
k(x) = (h \circ g \circ f)(x)
\]

\[
k(x) = h(g(f(x))) = h(g(x^2)) = h(2x^2) = 2x^2 + 3
\]

Decomposing Functions

Write \( h(x) = (2x^2 + 3)^3 \) as the composition of two simpler functions

\[
f(x) = 2x^2 + 3
\]

\[
g(x) = x^3
\]

\[
(g \circ f)(x) = g(f(x)) = g(2x^2 + 3) = (2x^2 + 3)^3 = h(x)
\]

\[
h(x) = (g \circ f)(x)
\]

\[
q(x) = x^2
\]

\[
r(x) = (2x + 3)^3
\]

\[
h(x) = (r \circ g)(x)
\]

Decomposing Functions

Write \( h(x) = |5 - x^3| \) as the composition of two simpler functions

\[
f(x) = 5 - x^3
\]

\[
g(x) = |x|
\]

\[
(g \circ f)(x) = g(f(x)) = g(5 - x^3) = |5 - x^3| = h(x)
\]

\[
h(x) = (g \circ f)(x)
\]

Decomposing Functions

Write \( h(x) = \sqrt{x^2 - 3x + 1} \) as the composition of two simpler functions

\[
f(x) = x^2 - 3x + 1
\]

\[
g(x) = \sqrt{x}
\]

\[
(g \circ f)(x) = g(f(x)) = g(x^2 - 3x + 1) = \sqrt{x^2 - 3x + 1} = h(x)
\]

\[
h(x) = (g \circ f)(x)
\]

Decomposing Functions

Write \( h(x) = \frac{1}{x^2 - 5} \) as the composition of two simpler functions

\[
f(x) = x^2 - 5
\]

\[
g(x) = \frac{1}{x}
\]

\[
(g \circ f)(x) = g(f(x)) = g(x^2 - 5) = \frac{1}{x^2 - 5} = h(x)
\]

\[
h(x) = (g \circ f)(x)
\]