Inverse Functions

\[ f(x) = 5x + 3 \quad g(x) = \frac{x - 3}{5} \]

\( (f \circ g)(x) = f(g(x)) = 5(g(x)) + 3 = 5\left(\frac{x - 3}{5}\right) + 3 = x - 3 + 3 = x \)

\( (g \circ f)(x) = g(f(x)) = \frac{f(x) - 3}{5} = \frac{5x + 3 - 3}{5} = \frac{5x}{5} = x \)

\[ f(g(x)) = x \quad \text{and} \quad g(f(x)) = x \quad f \text{ undoes } g \quad \text{and} \quad g \text{ undoes } f \]

\[ g(x) = f^{-1}(x) \quad \text{and} \quad f(x) = g^{-1}(x) \]

Finding Inverse Functions

Replace \( f(x) \) by \( y \) in the equation for \( f(x) \)

Interchange \( x \) and \( y \).

Solve for \( y \). If \( y \) is a function of \( x \), you’ve found the inverse, otherwise, there is not an inverse.

If you’ve found an inverse, replace \( y \) with \( f^{-1}(x) \)

Finding Inverse Functions

If \( f(x) = 5(x-1)^3 + 3 \), find \( f^{-1}(x) \)

\[ y = 5(x-1)^3 + 3 \]
\[ x = 5(y-1)^3 + 3 \]
\[ x - 3 = 5(y-1)^3 \]
\[ \frac{x - 3}{5} = (y-1)^3 \]
\[ \sqrt[3]{\frac{x - 3}{5}} = y - 1 \]
\[ \sqrt[3]{\frac{x - 3}{5}} + 1 = y \quad f^{-1}(x) = \sqrt[3]{\frac{x - 3}{5}} + 1 \]

Finding Inverse Functions

If \( f(x) = x^2 + 2 \), find \( f^{-1}(x) \)

\[ y = x^2 + 2 \]
\[ x = y^2 + 2 \]
\[ x - 2 = y^2 \]
\[ y^2 = x - 2 \]
\[ y = \pm \sqrt{x - 2} \]

Not a function, \( f(x) \) doesn't have an inverse

Finding Inverse Functions

Replace \( f(x) \) by \( y \)

Interchange \( x \) and \( y \).

Solve for \( y \).

Replace \( y \) with \( f^{-1}(x) \)

Invertible Functions are One-to-One
Invertible?

Graph the inverse

\[ f(x) = 5(x-1)^3 + 3 \]
\[ f^{-1}(x) = \sqrt[3]{\frac{x-3}{5}} + 1 \]