Dividing Polynomials

\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 \]

\[ f(x) = a_n (x - k_n)(x - k_{n-1}) \cdots (x - k_2)(x - k_1) \]

\[ 24 ÷ 3 = 8 \quad 24 = 8 \cdot 3 \]

\[ (x^2 - 5x - 6) ÷ (x + 1) = (x - 6) \]

\[ (x^2 - 5x - 6) = (x + 1) \cdot (x - 6) \]

How to divide two numbers

\[ 233 ÷ 6 = 38 \quad 5 \quad \frac{38}{6} \]

\[ \frac{233}{6} \quad 3 \quad 18 \quad 53 \]

\[ \frac{233}{6} \quad 53 \quad -48 \quad 5 \]

How to divide two polynomials

\[ (x^3 + 2x^2 + x^2 + 1) ÷ (x^2 - x + 1) \]

\[ \frac{x^2 + 2x + 2}{x^2 - x + 1} \]

\[ x^2 - x + 1 \]

\[ x^3 + 2x^2 + x^2 + 1 \]

\[ - (x^2 - x + 1) \]

\[ x^2 + x^2 + x^2 \]

\[ + 1 \]

\[ - (x^4 - x^2 + x^2) \]

\[ + 1 \]

\[ - (2x^3 - 2x + 1) \]

\[ 5 \]

\[ - (2x^3 - 2x + 1) \]

Dividing by \(x - k\)

Is 5 a root? (That means \(f(5) = 0\))
Is \(x - 5\) a factor?
If \(x - 5\) is a factor, then
\(x - 5\) will divide the polynomial with no remainder

Dividing by \(x - k\)

Is \(k\) a root? (That means \(f(k) = 0\))
Is \(x - k\) a factor?
If \(x - k\) is a factor, then
\(x - k\) will divide the polynomial with no remainder
Dividing by $x - k$

\[
x - k \mid \frac{ax^2 + (b + ka)x + (c + k(b + ka))}{(b + ka)x^2 + cx + d}
\]

\[
\begin{align*}
\text{Dividing by } x - k \\
\text{( remainder: } d + k(c + k(b + ka)))
\end{align*}
\]

The Remainder Theorem

- If you divide a polynomial $p(x)$ by $x - k$, the remainder is $p(k)$.
- If the remainder is 0, $p(k) = 0$ and $x - k$ is a factor.

### Example 1

Dividing by $x - k$

\[
(3x^2 - 2x - 3) ÷ (x - 3)
\]

- $3 \mid 1 -5 1 1$
- $3 -6 -15$
- $1 -2 -5 -14$

- $1x^2 - 2x - 5 \quad \text{R} \ -14$
- $x^2 - 2x - 5 \quad \text{R} \ -14$

### Example 2

Dividing by $x - k$

\[
(4x^3 - 2x^2 + 3x + 1) ÷ (x + 2)
\]

### Example 3

Dividing by $x - k$

\[
(4x^3 - 5x^2 + 2x - 3x + 1) ÷ (x - (-2))
\]

- $-2 \mid 4 -5 2 -3 1$
- $-8 16 -22 40 -74$
- $4 -8 11 -20 37 -73$

\[
233 ÷ 6 = 38 \quad \text{Remainder 5}
\]

\[
233 = 6 \cdot 38 + 5
\]

\[
p(x) ÷ (x - k) = q(x) \quad \text{Remainder } r
\]

\[
p(x) = (x - k)q(x) + r
\]

- If you divide a polynomial $p(x)$ by $x - k$, the remainder is $p(k)$.
- If the remainder is 0, $p(k) = 0$ and $x - k$ is a factor. 
Using the Remainder Theorem
If \( f(x) = -x^3 + 3x^4 + 21x^2 - 17 \), find \( f(-5) \)
\[ f(-5) = 5508 \]

\[
\begin{array}{c|ccccc}
-5 & -1 & 3 & 0 & 21 & 0 & -17 \\
 & & 5 & -40 & 200 & -1105 & 5525 \\
\hline
& -1 & 8 & -40 & 221 & -1105 & 5508
\end{array}
\]

Using the Remainder Theorem
If \( f(x) = 2x^4 + 7x^2 + 9x - 8 \), find \( f(-1) \)
\[ f(-1) = -8 \]

\[
\begin{array}{c|cccc}
-1 & 2 & 0 & 7 & 9 & -8 \\
 & & -2 & 2 & -9 & 0 \\
\hline
& 2 & -2 & 9 & 0 & -8
\end{array}
\]

Factoring Using Division
Factor \( x^3 - 2x^2 - x + 2 \) given that \(-1\) is a root
\[ x^3 - 2x^2 - x + 2 = (x + 1)(x^2 - 3x + 2) = (x + 1)(x - 2)(x - 1) \]

\[
\begin{array}{c|ccc}
-1 & 1 & -2 & -1 & 2 \\
 & & -1 & 3 & -2 \\
\hline
& 1 & -3 & 2 & 0
\end{array}
\]

Factoring Using Division
Factor \( 2x^3 - 3x^2 - 11x + 6 \) given that \( x + 2 \) is a factor
\[ 2x^3 - 3x^2 - 11x + 6 = (x + 2)(2x^2 - 7x + 3) = (x + 2)(2x - 1)(x - 3) \]

\[
\begin{array}{c|ccc}
-2 & 2 & -3 & -11 & 6 \\
 & & -4 & 14 & -6 \\
\hline
& 2 & -7 & 3 & 0
\end{array}
\]