Modeling Using Variation

“y varies directly as x”  \( y = kx \)

“y varies inversely as x”  \( y = \frac{k}{x} \)

“z varies jointly as x and y”  \( z = kxy \)

“z varies directly as the square of x and inversely as the cube root of y”  \( z = k\frac{x^2}{\sqrt[3]{y}} \)

Four steps:

1. Write an equation that describes the given English Statement.
2. Substitute the given pair of values into the equation in step one to find the value of \( k \).
3. Substitute the value of \( k \) into the equation from step 1.
4. Use the equation in step 3 to answer the problem’s question.

Modeling Using Variation

At a fixed speed, the distance you’ve traveled varies directly as the time you’ve been traveling. If you travel 225 miles in 3 hours, how far will you travel in 7 hours?

\[ d = kt \]

\[ 225 = k \cdot 3 \]

\[ 75 = k \]

\[ d = 75t \]

\[ d = 75(7) = 525 \text{ miles} \]

Modeling Using Variation

The time to travel a fixed distance at a constant speed varies inversely as the speed. If it takes you 45 minutes to get to Grandma’s house at 35 mph how long will it take you to get there at 21 mph?

\[ t = \frac{k}{s} \]

\[ 45 = \frac{k}{35} \]

\[ k = 1575 \]

\[ t = \frac{1575}{s} \]

\[ t = \frac{1575}{21} = 75 \text{ minutes} \]

Modeling Using Variation

The volume of a fixed amount of gas varies directly as the absolute temperature and inversely as the pressure. If a balloon has a volume of 1 liter when the pressure is 1 atm and the temperature is 293 K, what will the volume (to the nearest hundredth) be when the pressure is 2 atm and the temperature is 253 K?

\[ V = \frac{kT}{P} \]

\[ 1 = k \cdot \frac{293}{1} \]

\[ k = \frac{1}{293} \]

\[ V = \frac{1}{293} \cdot \frac{T}{P} \]

\[ V = \frac{1}{293} \cdot \frac{253}{2} = \frac{253}{586} \approx 0.43 \text{ liters} \]

Modeling Using Variation

The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. A wire of 720 feet with \( \frac{1}{4} \) in diameter has resistance of 1.5 ohms. Find the resistance for a 540 foot wire of the same type with a diameter that is half as big.

\[ R = k\frac{l}{d^2} \]

\[ 1.5 = k\frac{720}{\left(\frac{1}{4}\right)^2} \]

\[ k = \frac{1}{7680} \]

\[ R = \frac{\frac{540}{\left(\frac{1}{8}\right)^2}}{7680} = 4.5 \text{ ohms} \]