Inverse of an Exponential Function

\[ f(x) = b^x \]

If \( b > 1 \), \( f(x) = b^x \) is an increasing function.

If \( 0 < b < 1 \), \( f(x) = b^x \) is a decreasing function.

Every exponential function has an inverse.

Inverse of an Exponential Function

\[ f(x) = b^x \]

\( y = b^x \)

\( x = b^y \)

Stuck, algebraically, but not verbally.

"y is the exponent you put on \( b \) to get \( x \)."

\( f(x) = 2^x \)

\( f^{-1}(8) = ? \)  \( f \) inverse of 8 is the exponent you put on 2 to get 8.

\( 2^3 = 8 \quad 2^4 = 8 \)

\( f^{-1}(8) = 3 \)

logarithm: logos + arithmos

"reason (logic) number"

Inverse of an Exponential Function

\( y = \log_b x \)

"y is the logarithm base \( b \) of \( x \)."

\( \log_3 \frac{1}{27} = ? \quad 3^7 = \frac{1}{27} \quad 3^3 = 3^{-3} \quad \log_3 \frac{1}{27} = -3 \)

\( \log_{10} 4 = ? \quad 64^\frac{1}{3} = 4 \quad (4^3)^\frac{1}{3} = 4 \)

\( \log_{10} 4 = 1/3 \)

Some basic properties of logs

\( \log_b b = ? \quad b^1 = b \quad b^0 = 1 \quad \log_b 1 = 0 \)

\( \log_b b^x = ? \quad b^1 = b^1 \quad b^1 = b^1 \quad \log_b b^x = x \)

\( b^{\log_b x} = ? \quad b \) the exponent you put on \( b \) to get \( x \)

\( b^\log_b x = x \)
Domain of a log.

\[ g(x) = \log_b x \]

Since \( b > 0 \), there will only be an answer if \( x > 0 \)

\[ D_g = \{ x \mid x > 0 \} \]

\[ h(x) = \log_b (x^2 - 4) \]

\[ x^2 - 4 > 0 \]

\[ (x - 2)(x + 2) > 0 \]

\[ D_h = \{ x \mid x < -2 \text{ or } x > 2 \} \]

Inverse Properties of Logs

\[ \log b^r = x \quad \quad b^{\log_b x} = x \]

\[ f(x) = b^r \quad g(x) = \log_b x \]

\[ f(g(x)) = \log_b f(x) = \log_b b^r = x \]

\[ f(g(x)) = b^{r(x)} = b^{\log_b x} = x \]

\[ D_f = \{ x \mid x > 0 \} \]

\[ D_g = \{ x \mid x > 0 \} \]

\[ R_f = \{ x \mid x > 0 \} \]

\[ R_g = \{ x \mid x \text{ is a Real number} \} \]

\[ g(x) = f^{-1}(x) \quad f(x) = g^{-1}(x) \]

Graphing Logs

Graph \( f(x) = \log_b x \)

\( f(x) \) is the inverse of \( g(x) = 3^r \)

Graph \( f(x) = 2 - \log_2 x \)

\( f(x) \) is \( \log_2 x \) flipped vertically and shifted up 2.

Common and Natural Logs

"\( \log_{10} x \)" is abbreviated "\( \log x \)"

"\( \log_e x \)" is abbreviated "\( \ln x \)"

\[ \log 0.01 = ? \quad \log_{10} 0.01 = ? \quad 10^{-2} = 0.01 \]

\[ 10^2 = \frac{1}{100} \quad 10^{-2} = 0.01 \quad \log 0.01 = -2 \]

Common and Natural Logs

Graph \( f(x) = \ln(2 - x) \)

\( \ln(2 - x) = \log_e (2 - x) \)

\( f(x) \) is \( \log_e x \) shifted left 2 and flipped horizontally

If \( g(x) = \log_e x \)

and \( h(x) = g(x + 2) \),

\( f(x) = h(-x) = g(-x + 2) \)

\( \log_e x \) is the inverse of \( e^r \)

shift left 2.

flip horizontal