Properties of Logs-Product Rule

\[ \log_b x \text{ is the EXPONENT you put on } b \text{ to get } x \]

\[ b^x b^y = b^{x+y} \]

The exponent of a product is the sum of the exponents

The log of a product is the sum of the logs

\[ \log_b MN = \log_b M + \log_b N \]

\[ \log_b 15 = \log_b 3 \cdot 5 = \log_b 3 + \log_b 5 \]

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Properties of Logs-Quotient Rule

\[ \log_b x \text{ is the EXPONENT you put on } b \text{ to get } x \]

\[ \frac{b^x}{b^y} = b^{x-y} \]

The exponent of a quotient is the difference of the exponents

The log of a quotient is the difference of the logs

\[ \log_b \frac{M}{N} = \log_b M - \log_b N \]

\[ \log_b \frac{4}{7} = \log_b 4 - \log_b 7 \]

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Properties of Logs-PowerRule

\[ \log_b x \text{ is the EXPONENT you put on } b \text{ to get } x \]

\[ (b^x)^p = b^{xp} \]

The exponent of \( b^x \) to a power is the power times the exponent of \( b^x \)

The log of \( b^x \) to a power is the power times the log of \( b^x \)

\[ \log_b x^p = p \log_b x \]

\[ \log_b 7^3 = 3 \log_b 7 \]

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Expanding Logs

If \( \log_b 7 = A \) and \( \log_b 9 = G \),

Write \( \log_b 5103 \) in terms of \( A \) and \( G \)

\[ \log_b 9^7 \]

\[ = \log_b 9^7 + \log_b 7 \]

\[ = 3 \log_b 9 + \log_b 7 \]

\[ = 3G + A \]

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Condensing Logs

Write \( 3 \log_b x^2 + \log_b y^2 \) as a single logarithm.

\[ \log_b MN = \log_b M + \log_b N \]

\[ \log_b M + \log_b N = \log_b MN \]

\[ \log_b (x^2)^y + \log_b y^2 \]

\[ = \log_b x^6 + \log_b y^2 \]

\[ = \log_b x^6 y^2 \]
Be careful about domains

\( \log_b x^r = r \log_b x \)

\( \log_b x^2 = 2 \log_b x \)

\( \log_b x^2 \) is defined for every \( x \) except 0.

\( 2 \log_b x \) is only defined for \( x > 0 \)

The Change of Base Rule

Find \( \log_2 9 \) to the nearest thousandth.

\[ \log_b x = \frac{\log_{b'} x}{\log_{b'} b} \]

\[ \log_2 9 = \frac{\log_{10} 9}{\log_{10} 2} \]

\[ \log_2 9 = \frac{\ln 9}{\ln 2} \]

\[ \log_2 9 = 1.129 \]

Proof of the Product Rule

Let \( x = \log_b M \) and \( y = \log_b N \)

\( b^x = M \) and \( b^y = N \)

\[ MN = b^x b^y = b^{x+y} \]

\[ \log_b MN = x + y = \log_b M + \log_b N \]

Proof of the Quotient Rule

Let \( x = \log_b M \) and \( y = \log_b N \)

\( b^x = M \) and \( b^y = N \)

\[ \frac{M}{N} = \frac{b^x}{b^y} = b^{x-y} \]

\[ \log_b \frac{M}{N} = x - y \]

\[ \log_b \frac{M}{N} = \log_b M - \log_b N \]

Proof of the Power Rule

Let \( y = \log_b x^r \)

\( b^y = x^r \)

\[ b^{y/p} = x \]

\[ y/p = \log_b x \]

\[ y = p \log_b x \]

\[ \log_b x^r = p \log_b x \]
Proof of the Change of Base Rule

Let $y = \log_a x$

$b^y = x$

$\left(a^{\log_a b}\right)^y = x$

$a^y = b^x$

$y \log_a b = \log_a x$

$\log_a x \cdot \log_a b = \log_a x \cdot \log_a b$

$\log_a x = \log_a x \quad \log_a x = \frac{\log_a x}{\log_a b}$